Selection of SLR2000 Acquisition Parameters*

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Introduction and Summary

The SLR2000 autonomous and eyesafe satellite laser ranging system will acquire satellite targets at low elevation angles by correlation-aided detection of the time-of-arrival of a number of returned pulses. There is an optimum combination of the correlation parameter k (minimum number of pulse arrival times that must be correlated to declare a successful acquisition) and the frame time T_F (minimum time required to accumulate these correlations). This combination depends upon known quantities (e.g. the system hardware performance) and estimated ones (e.g. the two-way atmospheric path transmission).

In this paper we develop an analytic method to select values of k and T_F which simultaneously provide a high probability of detection (> 90%), and a low probability of false acquisition (< 1%), while allowing for significant uncertainty in the estimated quantities (~ ±25 %) such as the path transmission.

Acquisition with SLR2000

There are three areas of uncertainty during acquisition: the precise pointing angle to the target, the range / range rate of the target, and the expected signal amplitude.

Because of the expected system pointing errors, the initial angular uncertainty is ~ ± 80 μradians, while the beam size is constrained to ± 20 μradians to provide an adequate signal level. The NASA Goddard solution has been to implement a step spiral scan, centered on the most probable of the approximately ~ 17 locations. One dwells on each angular location only long enough to reliably acquire (> 90% probability), and to preclude false acquisition (< 1% probability per dwell time) --- this is a desired operating point. The first key parameter that arises in this approach is the dwell time per spot , also called the Frame Time, T_F.

Time-of-flight uncertainties arise since the target's range and range rate are imprecisely known, with the degree of uncertainty being dependent on both satellite altitude and zenith angle. The solution has been to implement a range gate (~ 200 nsec), and partition this range gate into time bins (~ 500 psec). These bin widths are adequate to compensate for both system timing and range rate uncertainty effects, i.e. over a given frame time all the signal pulse arrival times will cluster within one bin over the range gates included in the Frame, as shown in Figure 1. This is the basis for acquisition by correlation detection acquisition. The correlation parameter, k, describes this process: detect k pulses within the same time bin of the range gate, after viewing N_{RG} Range Gates, where N_{RG} = PRF x T_F, for PRF = the transmitter pulse repetition frequency.

* Supported in part by AlliedSignal Technical Services Corporation, and NASA Goddard Space Flight Center

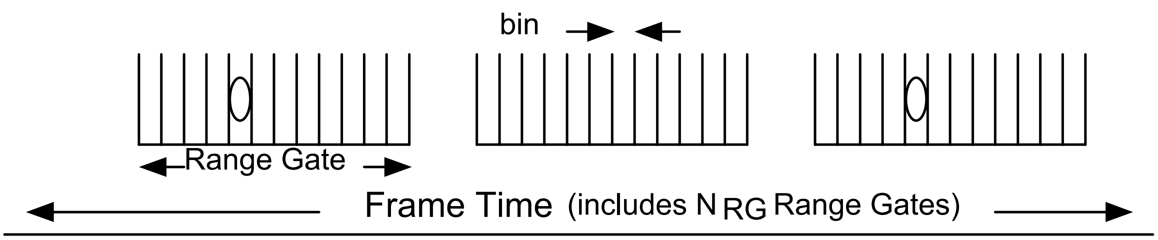

Figure 1. Correlation Detection Definitions and Example

The signal amplitude is uncertain because the two-way path transmission can only be inferred from day/night background, meteorology, and experience. k and T_F are dependent on the signal amplitude as well as the noise count rate, as seen below.

The challenge is to choose "best" values for k and T_F when initiating acquisition. The answer to this challenge forms the remainder of this paper. We start with the dependencies of the measures of performance (acquisition and false detection probabilities) on the system and correlation parameters.

As previously derived (cf. the EOO-authored documents in the Bibliography), the Probability of False Acquisition per Frame is given by

$$P_{FalseAcq} = 1 - \left\{ e^{-\overline{m}} \sum_{j=0}^{k-1} \frac{(\overline{m})^j}{j!} \right\}^{n_{bin}} = 1 - e^{-n_{bin}\left\{ \overline{m} - ln\left[\sum_{j=0}^{k-1} \frac{\overline{m}^j}{j!} \right] \right\}} \qquad (1)$$

where n_{bin} = number of time bins within a single time gate;

k = correlation parameter, the number of range gates over a frame which must have the same bin with a count in order to declare acquisition;

$\overline{m}$ = mean noise counts per bin over the frame

$$= \dot{n}_{pe}\, t_{bin} \left(PRF\ T_F\right); \qquad (2)$$

for $\dot{n}_{pe}$ = total noise count rate (sec^{-1}),

t_{bin} = bin width (sec).

For SLR2000, during the minimum bin width = 500 psec, and the PRF = 2000 Hz, so

$$\overline{m} = 10^{-6}\, \dot{n}_{pe} T_F \ . \qquad (3)$$

From the same references, the probability of signal detection per frame is

$$P_D = 1 - e^{-N_t} \sum_{j=0}^{k-1} \frac{(N_t)^j}{j!} \qquad (4)$$

where N_t = The total mean number of correlated pe's detected in the same bin.

During the stressing SLR2000 acquisition function,

$$N_T \sim N^s_{pe} \quad (5)$$

for N^s_{pe} = total number of signal pe's detected during the frame,

and $N^s_{pe} = n^s_{pe}\ (PRF\ T_F)$, (6)

where n^s_{pe} = mean signal pe count per pulse, or Range Gate interval.

n^s_{pe} is derived/estimated from the Range Equation, with the major dependencies:

n^s_{pe} = {System Hardware Properties}{ Range^{-4}}{two-way path transmission}.

Since the two way path transmission is only indirectly estimated from other observables, the expected value of n^s_{pe} is also uncertain.

For acquisition with the SLR2000 (with the typical signal count rate of ~ 10 per second and $\overline{m}$ < 1, so that Equation 5 holds),

$$N_t = 2000\ n^s_{pe}\ T_F . \quad (7)$$

Selecting Values of k and T_F

Conceptually the approach is to specify a large probability of detection and a small probability of false acquisition, and to then use the inverted probability equations, i.e. solve for the dependence of k and T_F on each other, as well as the two probabilities, the signal and the noise levels.

Heuristically, for the False Acquisition Probability,

$$P_{FalseAcq} = 1 - e^{-n_{bin}\left\{\overline{m} - ln\left[\sum_{j=0}^{k-1} \frac{\overline{m}^j}{j!}\right]\right\}} \Longrightarrow k = f(P_{False\ Acq}, n_{bin}, \overline{m}\ (T_F)); \quad (8)$$

and for the Probability of Detection,

$$P_D = 1 - e^{-N_t} \sum_{j=0}^{k-1} \frac{(N_t)^j}{j!} \Longrightarrow k = f(P_D, N_t(T_F)). \quad (9)$$

Given the success of the above quasi-inversions, we can then choose combinations of k and T_F which satisfy both equations, assuring us that the operating point will satisfy both probability criteria

Noise / False Acquisition Equation

Since the algebra involved precludes a direct inversion approach, we invert the equation by first evaluating it for a given value of the $P_{FalseAcq}$, and then tabulating the resulting relation between k and $\overline{m}$ for a given number of bins. We curve fit the k/$\overline{m}$ relation, substitute Equation 3 into the "fit" equation, to obtain a k / T_F relation.

The expression for the False Acquisition Probability, Equation 1, is evaluated in Figure 2 for the 1% value of the Probability, with $\overline{m}$ as a function of the number of bins, and k as a parameter.

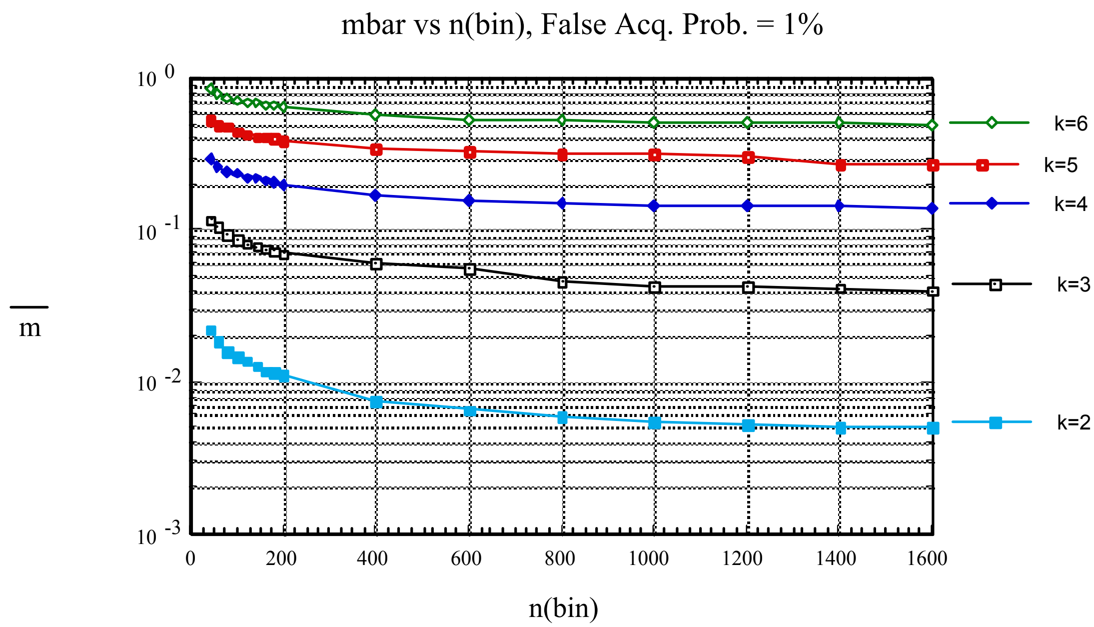

Figure 2. Relationship of the mean noise counts per bin over the frame, and the number of bins per range gate, with k as a parameter, for a 1% $P_{FalseAcq}$ per frame.

We see from the figure that $\overline{m}$ has a weak dependence on the number of bins, except for n(bins) less than ~400. However, the value of $\overline{m}$ that will provide the desired low probability of false acquisition increases by orders of magnitude as k is increased from k = 2 to k = 6.

Table 1 lists the k/$\overline{m}$ pairs in Figure 2 at the 400 bin line, for k = 2, 3, 4, 5 and 6. We then perform a curve fit for the values in this Table.

Table 1. Noise and correlation parameter relationship for a 1% False Acquisition Probability and 400 time bins per range gate.

k	$\overline{m}$ (n_{bin} = 400)
2	0.0075
3	0.0593
4	0.1694
5	0.3459
6	0.5702

The result of this curve fit is

$$k = 36.285\overline{m}^3 - 40.682\overline{m}^2 + 18.558\overline{m} + 1.992 \qquad \text{(400 bins/1\%)} \qquad (10)$$

Substituting Equation 3 into Equation 10 we arrive an expression relating the correlation parameter, the frame time, and the noise count rate, for 400 bins.

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$$k = 3.63\left(10^{-17}\right)\left[\dot{n}_{pe}\,T_F\right]^3 - 4.07\left(10^{-11}\right)\left[\dot{n}_{pe}\,T_F\right]^2 + 1.86\left(10^{-5}\right)\left[\dot{n}_{pe}\,T_F\right] + 1.992$$

[400 bins/1%] (11)

Equation 11 (and the appropriate one for 200 bins) is plotted in Figure 3. We see from the figure that the required values of k/T_F are only weakly dependent on the number of bins, as expected, but that high noise counts and long frame times demand very high k values. Long frame times may be impractical because of the number of spots that have to be scanned in the SLR200 to cover the initial angular uncertainty.

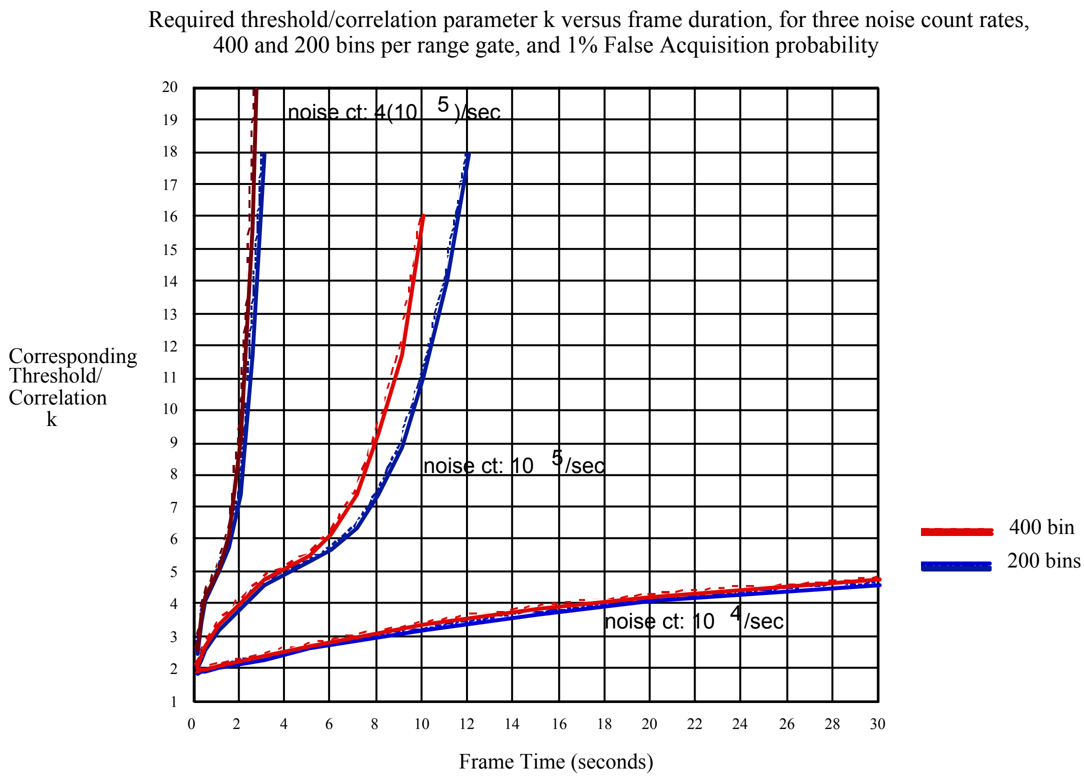

Figure 3. Evaluation of the k/T_F relationship for a 1% $P_{FalseAcq}$, for 400 and 200 bins per range rate. The noise count rates encompass both day and night operations

Signal Detection

In Table 2 we evaluate the probability of correlated signal detection, Equation 4. The table lists the detection probabilities as a function of the k parameter and the total mean number of signal photo-electrons detected per frame.

Table 2. Probability of Detection for the mean numbers of signal pe detected per frame and the correlation parameter k.

Mean # of Signal pe's detected per Frame N_{pe}^{s}	$P_k(\geq 2)$	$P_k(\geq 3)$	$P_k(\geq 4)$	$P_k(\geq 5)$	$P_k(\geq 6)$	$P_k(\geq 7)$	$P_k(\geq 8)$
1	0.264	0.080	0.02				
2	0.594	0.323	0.143				
3	0.80	0.577	0.353				
4	0.908	0.762	0.567				
5	0.96	0.875	0.734				
6	0.983	0.938	0.849				
7	0.993	0.970	0.918	0.827			
8	0.997	0.986	0.958	0.899	0.809		
9					0.884		
10					0.93	0.87	
11						0.92	0.857
12							0.91

We select the highlighted values, which are ~ 90% probabilities, and plot them in Figure 4, along with an appropriate curve fit.

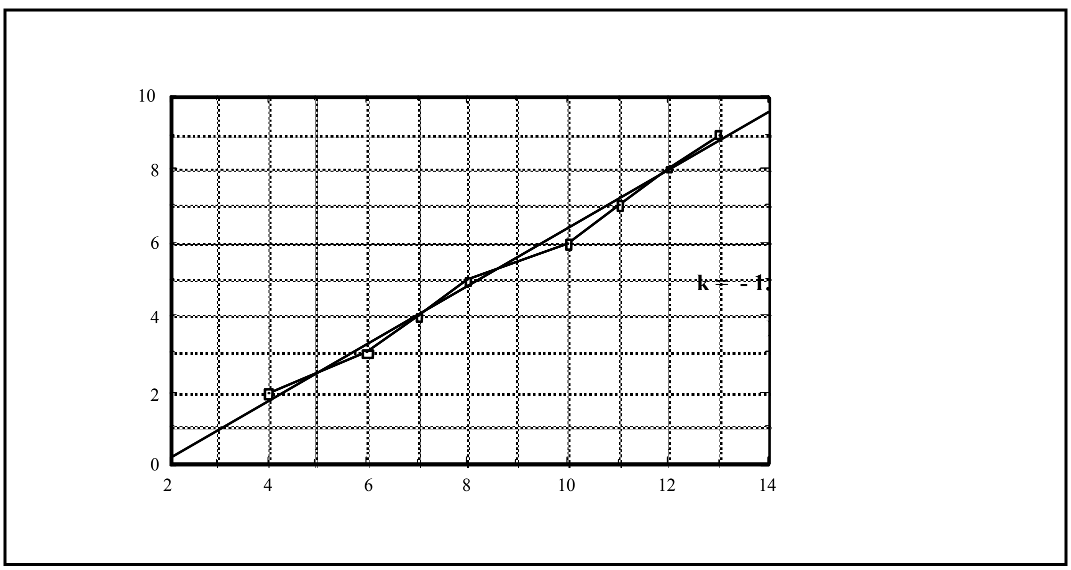

Figure 4. Correlation parameter versus the mean number of signal pe's detected per frame, for ~ 90% Probability of Detection

The curve fit result is given by $k = -1.4 + 0.78\ N_{pe}^{s}$. (90%) (12).

After we insert Equation 7 into Equation 12, we have a second relationship between k and T_F, with the mean signal level per pulse as the only other dependence.

$$k = -1.4 + 1560\ n_{pe}^{s}\ T_F,\quad (90\%). \qquad (13)$$

We have two equations relating k and T_F with the estimated signal and measurable noise rate parameters. Values of k / T_F that satisfy both equations will also meet both probability criteria, and be acceptable operating points.

Evaluation

Based on MODTRAN estimates for the path transmission and the background (cf. Bibliography), we take the realistic test values listed in Table 3 during acquisition.

Table 3. Test values for evaluation

Case #	1 (Day)	2 (Night)
PRF (kHz)	2	2
t_{bin} (pico-sec)	500	500
n_{bin}	400	400
n^s_{pe}	0.005	0.005
$\dot{n}^n_{pe}$ (per second)	200,000	5,000

We use the values in Table 3 to evaluate Equations 3 and 7 and derive the mean signal and noise levels per bin during the frame as listed in Table 4.

Table 4. Mean signal counts per frame, and mean noise counts per bin per frame.

Case #	1 (Day)	2 (Night)
N^s_{pe}	10 T_F	10 T_F
$\overline{m}$	0.2 T_F	0.005 T_F

The Table 3 values and Equations 11 and 13 lead to the results in Figure 5.

As an aid to interpretation, we have cross-hatched the allowable domain for Case 1, daytime, i.e. any combination of k and T_F values in the indicated domain will result in a better than specified performance, exceeding > 90% Detection Probability while providing < 1% of False Detection Probability per frame.

For purposes of comparison, we evaluate the exact probability equations for the k/T_F combination in the lowest corner of the cross-hatched region and find:

k = 4 $T_F = 0.7$ seconds

$N^s_{pe} = 7$ $\overline{m} = 0.14$ $P_D = 92.52\%$ $P_{FA} = 0.57\%$

Sensitivity Example

If the uncertainty of the expected signal level is ± 25%, one could instead choose to operate at k = 5. Since for Test Case #1,

$$k = -1.4 + 1560\, n^s_{pe} T_F, \tag{14}$$

if $n^s_{pe} = 0.00375$, instead of 0.005, then $T_F = 1.1$ seconds will satisfy both Probabilities.

Correlation Parameter vs Frame Time for 90% Signal Detection Probability (at 0.005 signal pe/pulse)
and 1% False Acquisition Probabilty, two noise count rate Cases

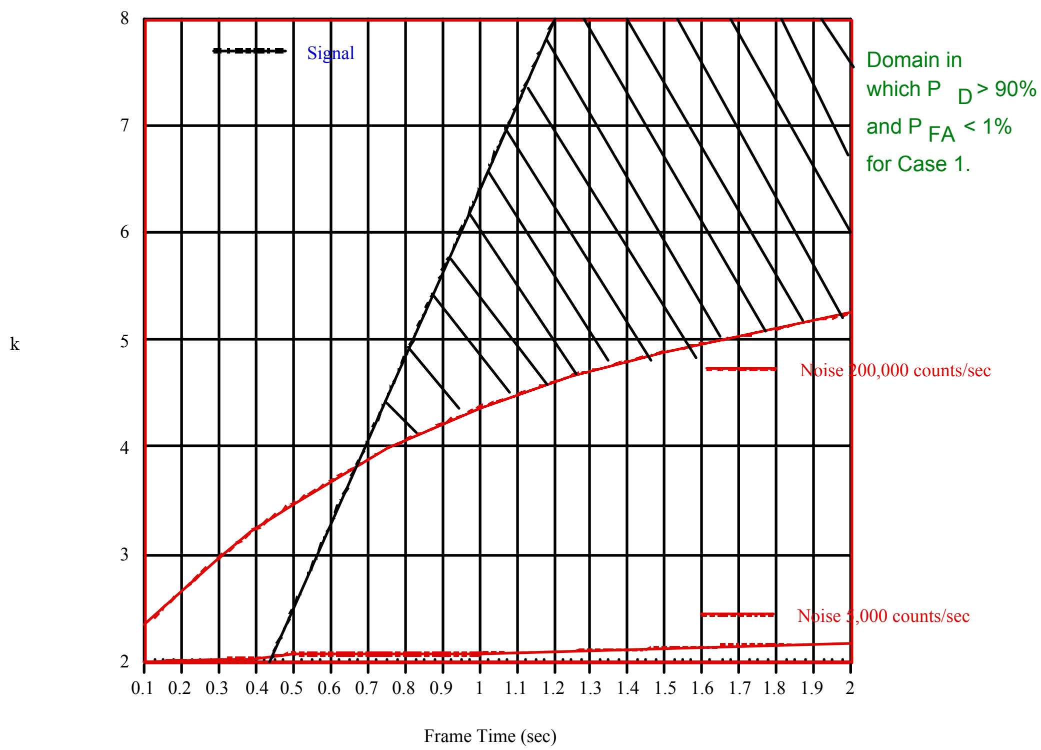

Figure 5. Operating region, satisfying both high signal detection probability and low false acquisition probability per frame.

Conclusion

This approach provides a precise technique for selecting the system set-up parameters for acquisition in the normal situation where a high probability of signal detection and a low probability of false acquisition are simultaneously desired. It also guarantees that a moderate miss-estimate of the signal level need not degrade performance, if the k / T_F pair are conservatively selected..

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